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PREDICTION OF FUNDAMENTAL FREQUENCY OF INITIALLY IN-PLANE-LOADED MODERATELY THICK CIRCULAR PLATES

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1. INTRODUCTION

A simple design formula to predict the fundamental frequency of initially loaded structural elements (either by axial compressive loads in the case of slender columns or by in-plane compressive loads in the case of thin square plates) has been presented by one of the authors [1–3]. It has been shown in these references that the formula gives exact results when the mode shapes of buckling and free vibration of load free and initially loaded structural elements are the same. Otherwise, the formula is approximate and the order of approximation is determined by how much these three mode shapes differ with each other. From references [1–3], based on the results, it can be seen that for all the practical situations, these three mode shapes are approximately the same, within engineering accuracy.

In the present paper, the efficacy of the proposed formula is studied through the example of the prediction of the fundamental frequency of moderately thick circular plates, where, the effects of shear deformation and rotatory inertia have to be included. Both simply supported and clamped boundary conditions for the circular plates are considered. The numerical results obtained from the proposed formula, when compared to those obtained from the finite element method (FEM) [4] show the usefulness of the formula in quickly predicting the fundamental frequencies of moderately thick circular plates and the design engineers can use this information for the initial design phase effectively.

In what follows the derivation of the design formula is briefly presented for completeness even though given in detail in reference [3].

2. BRIEF FORMULATION

For predicting the initially loaded free vibration behaviour of structural elements like bars, beams, plates, shell panels, etc. following the analyses methods like Rayleigh–Ritz, weighted residual, finite element and so on, the final matrix equation can be obtained in the form

$$[K]\{\delta\} - \lambda[G]\{\delta\} - \lambda_f[M]\{\delta\} = 0, \quad (1)$$

where, in the structural mechanics terminology, $[K], [G]$ and $[M]$ may be called as the system stiffness, geometric stiffness and mass matrices, respectively, and for a circular

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plate, λ is the compressive load parameter (defined as $\lambda = N_r a^2/D$) and λ_f is the initially loaded frequency parameter (defined as $\lambda_f = \rho\omega^2 a^4/D$) and $\{\delta\}$ is the eigenvector . 'a' is the radius of the circular plate, D is the plate fluxural rigidity, $[= Et^3/12(1 - v^2)]$, N_r is the radial edge compressive load per unit length, ρ is the mass density per unit area, ω is circular frequency of initially loaded plate, t is the plate thickness and v is the Poisson ratio. It may be noted here that the matrices $[K]$ and $[M]$ contain the effects of shear deformation and rotatory inertia, respectively.

Two degenerate cases can be obtained from equation (1), namely load-free vibration problem by neglecting the second term (then $\lambda_f = \lambda_{f0}$) and buckling problem of the circular plates by neglecting the third term (then $\lambda = \lambda_b$) , where, λ_{fo} is the load-free frequency parameter (defined as, $\lambda_f = \rho\omega_0^2 a^4/D$, ω_0 being the circular frequency of the load-free plate) and λ_b is the critical load parameter (defined as, $\lambda_b = N_{r\,cr}\, a^2/D, N_{r\,cr}$ being the critical radial compressive load).

Assuming that the mode shapes of load-free vibration, initially loaded vibration and buckling problems are the same, we get

$$[K]\{\delta\} - \frac{\lambda}{\lambda_b}[K]\{\delta\} - \frac{\lambda_f}{\lambda_{f0}}[K]\{\delta\} = 0. \tag{2}$$

From equation (2), we get

$$\frac{\lambda}{\lambda_b} + \frac{\lambda_f}{\lambda_{f0}} = 1. \tag{3}$$

From equation (3), knowing λ, λ_b and λ_{f0} one can compute the frequency parameter λ_f of the initially loaded circular plate.

3. NUMERICAL EVALUATION

The effectiveness of the formula derived in the previous section is demonstrated by applying it to the vibration problem (predicting the fundamental frequency parameter λ_f) of a moderately thick uniform circular plates subjected to uniform compressive load N_r at the edge (Figure 1). The plate considered is either simply supported or clamped. For the sake of comparison, the results obtained by FEM [4] for both λ_f and buckling load parameter λ_b are taken. For FEM, the plate is idealized with eight annular ring elements of equal width which give the value of λ_f and λ_b up to four significant figures accurately.

The buckling load parameter λ_b and the load-free frequency parameter λ_{f0} of simply supported and clamped moderately thick circular plates obtained from FEM [4] are given in Table 1 for various values of t/a.

Table 2 gives the values of λ_f of a simply supported moderately thick circular plate obtained by both the formula and FEM. It can be seen that the agreement of the present results with those obtained by FEM are excellent for all the parameters considered, which in turn confirm the assumption that the mode shapes of load-free vibration (fundamental frequency), buckling and initially loaded vibration (fundamental frequency) are the same.

Similar results are given in Table 3 for a clamped moderately thick circular plate. The present results agree well with those obtained by FEM except for the extreme values of the thickness parameter, say 0·2, and initial load parameter, say 0·8, where the error is of the order of 4%. This suggests that the assumption made about the mode shapes is slightly violated for the extreme cases of the thickness and initial load parameters but are still within the engineering accuracy.

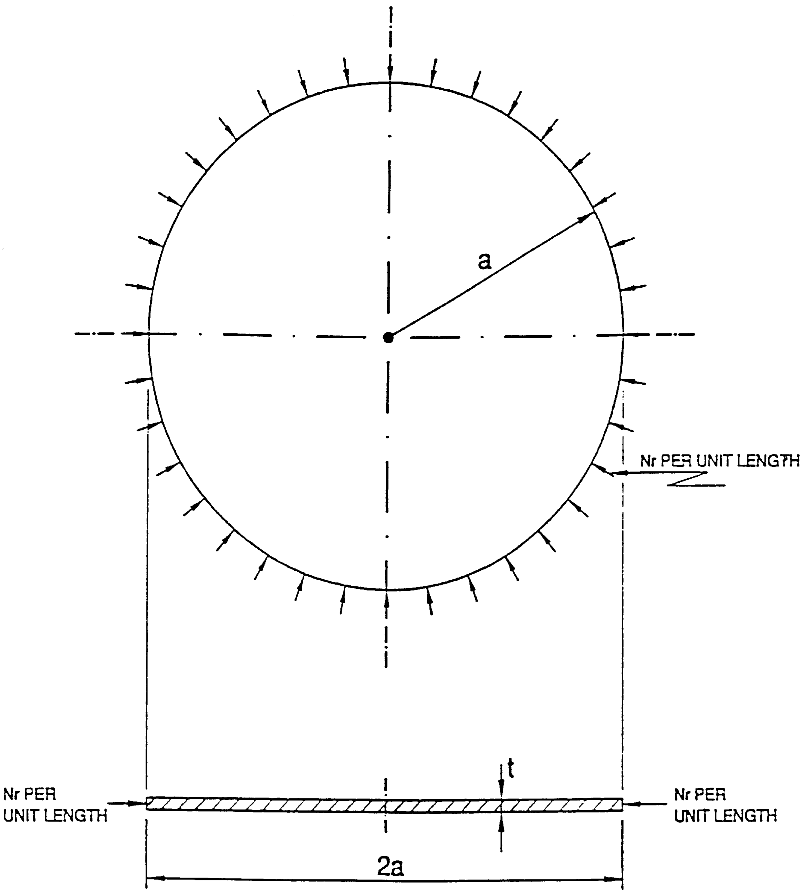

Figure 1. A uniform circular plate subjected to a uniform compressive load at the edge.

TABLE 1

Load-free frequency and buckling load parameters of moderately thick circular plates

T/a	Simply supported		Clamped	
	λ_{f0}	λ_b	λ_{f0}	λ_b
0·001	24·36	4·20	104·36	14·68
0·05	24·25	4·19	102·96	14·53
0·10	23·95	4·15	99·00	14·09
0·15	23·47	4·09	93·08	13·42
0·20	22·84	4·01	85·98	12·57

4. CONCLUDING REMARKS

The simple design formula proposed by one of the authors to predict the fundamental frequency of initially loaded structural elements, when their load-free fundamental frequency and buckling loads are known, is applied for predicting the fundamental

Table 2

Comparison of frequency parameter (λ_f) of initially loaded, simply supported, moderately thick circular plates

T/a	$\lambda/\lambda_b = 0{\cdot}2$			$\lambda/\lambda_b = 0{\cdot}4$			$\lambda/\lambda_b = 0{\cdot}6$			$\lambda/\lambda_b = 0{\cdot}8$		
	FEM	Formula	% change	FEM	Formula	% change	FEM	Formula	% change	FEM	Formula	% change
0·001	19·49	19·49	0·00	14·62	14·62	0·00	9·75	9·75	0·00	4·87	4·87	0·00
0·05	19·40	19·40	0·00	14·55	14·55	0·00	9·70	9·70	0·00	4·85	4·85	0·00
0·10	19·16	19·16	0·00	14·37	14·37	0·00	9·58	9·58	0·00	4·79	4·79	0·00
0·15	18·78	18·78	0·00	14·09	14·08	−0·07	9·39	9·37	−0·21	4·63	4·60	−0·65
0·20	18·28	18·27	−0·055	13·70	13·68	0·15	9·14	9·11	−0·33	4·54	4·50	−0·89

Table 3

Comparison of frequency parameter (λ_f) of initially loaded, clamped moderately thick circular plates

T/a	$\lambda/\lambda_b = 0{\cdot}2$			$\lambda/\lambda_b = 0{\cdot}4$			$\lambda/\lambda_b = 0{\cdot}6$			$\lambda/\lambda_b = 0{\cdot}8$		
	FEM	Formula	% change	FEM	Formula	% change	FEM	Formula	% change	FEM	Formula	% change
0·001	83·93	83·49	−0·53	63·32	62·62	−1·11	42·48	41·74	−1·76	21·39	20·87	−2·49
0·05	82·82	82·37	−0·55	62·41	61·78	−1·12	41·94	41·18	−1·84	21·13	20·59	−2·62
0·10	79·69	79·20	−0·62	60·17	59·40	−1·31	40·42	39·60	−2·07	20·38	19·80	−2·93
0·15	74·99	74·46	−0·71	56·69	55·85	−1·51	38·13	37·23	−2·40	19·25	18·62	−3·42
0·20	69·32	68·76	−0·81	52·47	51·57	−1·73	35·34	34·38	−2·79	17·88	17·19	−4·02

frequency of initially loaded moderately thick circular plates, where the effects of shear deformation and rotatory inertia are to be included. The numerical results presented show the effectiveness of the simple design formula, even when the secondary effects like shear deformation and rotatory inertia are included. The authors believe that the present formula is of immense use to the design engineers during the initial design phase.

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